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VISIBILITY GRAPHS AND PRECURSORS OF STOCK CRASHES

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Based on the network paradigm of complexity, a systematic analysis of the dynamics of the largest stock markets in the world has been carried out in the work. According to the algorithm of the visibility graph, the daily values of stock indices are converted into a network, the spectral and topological properties of which are sensitive to the critical and crisis phenomena of the studied complex systems. It is shown that some of the spectral and topological characteristics can serve as measures of the complexity of the stock market, and their specific behaviour in the pre-crisis period is used as indicators-precursors of crisis phenomena. The influence of globalization processes on the world stock market is taken into account by calculating the interconnection (multiplex) measures of complexity, which modifies in some way, but does not change the fundamentally predictive possibilities of the proposed indicators-precursors.

Keywords: *stock markets, graph theory, complex networks, visibility graph, spectral and topological analyzes, complexity measures, multiplex systems, financial system crashes.*

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Виходячи з мережної парадигми складності, у роботі проведено системний аналіз динаміки найбільших фондових ринків світу. За алгоритмом графа видимості щоденні значення фондових індексів перетворено у мережу, спектральні і топологічні властивості якої чутливі до критичних і кризових явищ досліджуваних складних систем. Показано, що деякі із спектральних і топологічних характеристик можуть слугувати мірами складності фондового ринку, а їх специфічна поведінка у передкризовий період використовуватись у якості індикаторів-передвісників кризових явищ. Вплив процесів глобалізації на світовий фондовий ринок враховано шляхом розрахунку міжмережних (мультиплексних) мір складності, які певним чином модифікують, але не змінюють принципово прогнозних можливостей запропонованих індикаторів-передвісників.

Ключові слова: *фондові ринки, теорія графів, складні мережі, граф видимості, спектральний і топологічний аналізи, міри складності, мультиплексні системи, крахи фінансових систем.*

ГРАФЫ ВИДИМОСТИ И ПРЕДВЕСТНИКИ ФОНДОВЫХ КРАХОВ

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В работе проведен системный анализ динамики крупнейших фондовых рынков мира, базируясь на сетевой парадигме сложности. Согласно алгоритму графа видимости ежедневные значения фондовых индексов преобразованы в сеть, спектральные и топологические свойства которой чувствительны к критическим и кризисным явлениям исследуемых сложных систем. Показано, что некоторые из спектральных и топологических характеристик могут служить мерами сложности фондового рынка, а их специфическое поведение в предкризисный период использоваться в качестве индикаторов-предвестников кризисных явлений. Влияние процессов глобализации на мировой фондовый рынок учтено путем расчета межсетевых (мультиплексных) мер сложности, которые определенным образом модифицируют, но не меняют принципиально прогнозных возможностей предложенных индикаторов-предвестников.

Ключевые слова: *фондовые рынки, теория графов, сложные сети, граф видимости, спектральный и топологический анализы, меры сложности, мультиплексные системы, крахи финансовых систем.*

JEL Classification: F37, C69

Introduction

Data from both fundamental and technical analysis indicate that there are enough reasons for the recession now. Among the main ones are the threat of further collapse in the financial markets, the trade war with China, and the consequences of a long shutdown. Analysts also note the weakening of the positive impact of fiscal incentives.

Participants in the debt market also bet on a rapid recession. The US yield curve took the form of a short-term inversion, signaling the likelihood of a downturn in the near future. This indicator is the most accurate and almost did not give false predictions in the history. Also, the danger isthat China and Saudi Arabia, the largest owners of American public debt, facing economic and budgetary problems, will simply be forced to sell US government bonds. It will almost inevitably trigger a massive escape of investors from the US public debt.

As for the stock market, after the 2008 crisis, the US market (its largest segment) recovered to its mid-2000 levels, when the Dow Jones index approached around 10500 points in October 2010. Since then, by January 2018, Dow Jones has reached its maximum level about 26600, having rolled over 8 years approximately in 2,5 times. The current virtually continuous growth — one of the longest in the history of the US stock market. For comparison: for the period from the beginning of 1928 to September 1929, the Dow Jones index rose from the level of 190 to 382, which is almost double. At the same time, the rally before the crisis in 2008 was much more modest: an increase from the level of 10000 (September 2005) to the maximum July 2007 about 13000 was only 1.3 times. The technical analysis shows a significant probability of continuing the fall of the stock market and the development of a crisis situation.

According to a recent report by experts from the World Economic Forum on Global Risk Factors in 2019, the main ones are the economic confrontation between the largest countries and the achievement of the pivotal pace of global economic growth [1].

One of the most prestigious anti-crisis management experts Nouriel Roubini predicts the global financial crisis in 2020 [2]. In his view, there are at least ten reasons for this, the main of which, in addition to the above, is the excessive level of credit in many developing countries and in some developed countries; excessive use of high frequency/algorithmic stock trading will increase the likelihood of a very sharp collapse of the markets (flash crash).

Therefore, in the face of a possible recession, it is important to timely carry out a forward-looking analysis of financial markets, to identify and test indicators of likely crises with a view to their early prevention.

The doctrine of the unity of the scientific method states that for the study of events in socio-economic systems, the same methods and criteria are applicable as in the study of natural phenomena. Significant success was achieved within the framework of interdisciplinary approaches and the theory of self-organization — synergetics, which according to the classification of G. Malinetsky [3] is on the verge of a new paradigm.

The modern paradigm of synergetics is a complexity paradigm. In the paradigm of complexity it is possible to investigate, based on the methods of mathematical modeling, data of natural sciences and interdisciplinary approaches, and set very deep questions [4]. In the framework of the complexity paradigm it became apparent that we should move from well-studied systems and processes, taking into account the minimal number of new entities that are characteristic of the social sciences or the humanities. Apparently, one of these entities is the bonds, that is, what characterizes the interaction of the elements that are part of the system, that makes parts of the whole. The set of these links is called network.

The new interdisciplinary study of complex systems, known as the complex networks theory, laid the foundation for a new network paradigm of synergetics [3]. He studies the characteristics of networks, taking into account not only their topology, but also statistical properties, the distribution of weights of individual nodes and edges, the effects of information dissemination, robustness, etc. [5–8]. Complex networks include electrical, transport, information, social, economic, biological, neural and other networks [9–11]. The network paradigm has become dominant in the study of complex systems since it allows you to enter new quantitative measures of complexity not existing for the time series [12]. Moreover, the network paradigm provides adequate support for the core concepts of Industry 4.0 [13].

Statement of the research task

Previously, we introduced various quantitative measures of complexity for individual time series [14, 15].

Significant advantage of the introduced measures is their dynamism, that is, the ability to monitor the time of change in the chosen measure and compare with the corresponding dynamics of the output time series. This allowed us to compare the critical changes in the dynamics of the system, which is described by the time series, with the characteristic changes of concrete measures of complexity. It turned out that quantitative measures of complexity respond to critical changes in the dynamics of a complex system, which allows them to be used in the diagnostic process and prediction of future changes. In [15], we introduced network complexity measures and adapted them to study system dynamics. But networks are rarely isolated. Therefore, it is necessary to take into account the interconnection interaction, which can be realized within the framework of different models [16]. We will consider it by simulating so-called multiplex networks, the features of which are reduced to a fixed number of nodes in each layer, but they are linked by different bonds [16].

Methods of converting time series into graphs

Most complex systems inform their structural and dynamic nature by generating a sequence of certain characteristics known as time series. In recent years, interesting algorithms for the transformation of time series into a network have been developed, which allows to extend the range of known characteristics of time series even to network ones. Recently, several approaches have been proposed to transform time sequences into complex network-like mappings. These methods can be conventionally divided into three classes [17]. The first is based on the study of the convexity of successive values of the time series and is called visibility graph (VG) [17, 19].

The second analyzes the mutual approximation of different segments of the time sequence and uses the technique of recurrent analysis [17]. The recurrent diagram reflects the existing repetition of phase trajectories in the form of a binary matrix whose elements are units or zeros, depending on whether they are close (recurrent) with given accuracy or not, the selected points of the phase space of the dynamic system. The recurrence diagram is easily transformed into an adjacency matrix, on which the spectral and topological characteristics of the graph are calculated [15, 18].

Finally, if the basis of forming the links of the elements of the graph is to put correlation relations between them, we obtain a correlation graph [15, 17]. To construct and analyze the properties of a correlation graph, we must form a adjacency matrix from the correlation matrix. To do this, you need to enter a value which, for the correlation field, will serve as the distance between the correlated agents. Such a distance may be dependent on the ratio of the correlation C_{ij} value $x(i, j)=\sqrt{2(1-C_{ij})}$. So, if the correlation coefficient between the two assets is significant, the distance between them is small, and, starting from a certain critical value x_{cr}, assets can be considered bound on the graph. For an adjacency matrix, this means that they are adjacent to the graph. Otherwise, the assets are not contiguous. In this case, the binding condition of the graph is a prerequisite.

The main purpose of such methods is to accurately reproduce the information stored in the time series in an alternative mathematical structure, so that powerful graph theory tools could eventually be used to characterize the time series from a different point of view in order to overcome the gap between nonlinear analysis of time series, dynamic systems and the graphs theory.

The use of the complexity of recurrent networks to prevent critical and crisis phenomena in stock markets has been considered by us in a recent paper [18]. Therefore, in this paper we will focus on algorithms of the VG and multiplex VG (MVG).

The algorithm of the VG is realized as follows [19]. Take a time series $Y(t)=[y_1, y_{2,},..., y_n]$ of length *n*. Each point in the time series data can be considered as a vertex in an associative network, and the edge connects two vertices if two corresponding data points can «see» each other from the corresponding point of the time series (Fig. 1). Formally, two values y_a of the series (at a point in time t_a) and y_b (at a point in time t_b) are connected, if for any other value (y_c,t_c), which is placed between them (that is, $t_a < t_c < t_b$), the condition is satisfied:

$$y_c < y_a + (y_b - y_a)\frac{t_c - t_a}{t_b - t_a}. \quad (1)$$

Note that the visibility graph is always connected by definition and also is invariant under affine transformations, due to the mapping method.

An alternative (and much simpler) algorithm is the horizontal visibility graph (HVG) [20], in which a connection can be established between two data points *a* and *b*, if one can draw a horizontal line in the time series joining them that does not intersect any intermediate dataheight y_c by the following geometrical criterion: $y_a, y_b > y_c$ for all *c* such that $t_a < t_c < t_b$).

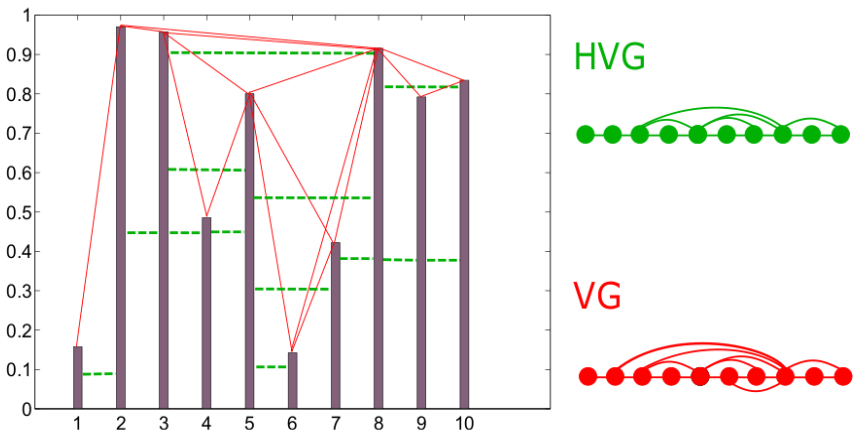

Fig. 1. Illustration of constructing the visibility graph (red lines) and the horizontal visibility graph (green lines)

In multiplex networks, there are two tasks: (1) turn separate time series on the network for each layer; (2) connect the intra-loop networks to each other. The first problem is solved within the framework of the standard algorithms described above. For interlayer interactions we use modified algorithm of VG. In this case, the normalized individual points of the time series are mutually visible, if (as in the case of a single row) the above conditions are fulfilled.

For multiplex networks, the algorithm of the MVG for the three layers is presented in Fig. 2.

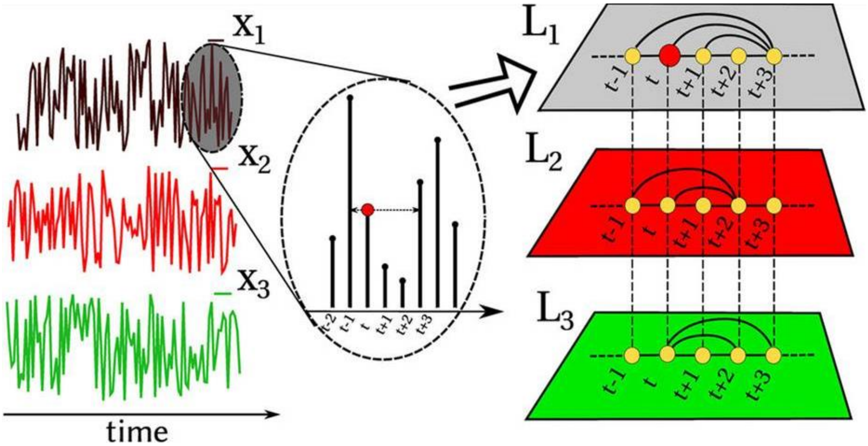

Fig. 2. Scheme for forming bonds between three layers of the multiplex network [28]

For constructed graph methods described above, one can calculate spectral and topological properties.

Spectral and topological graph properties

Spectral theory of graphs is based on algebraic invariants of a graph — its spectra. The spectrum of graph G is the set of eigenvalues of a matrix $S_p(G)$ corresponding to a given graph. For a adjacency matrix A of a graph, there exists an characteristic polynomial $|\lambda I - A|$, which is called the characteristic polynomial of a graph $P_G(\lambda)$. The eigenvalues of the matrix A (the zeros of the polynomial $|\lambda I - A|$) and the spectrum of the matrix A (the set of eigenvalues) are called respectively their eigenvalues and the spectrum of graph G. The eigenvalues of the matrix A satisfy the equality $A\bar{x} = \lambda\bar{x}$ ($\bar{x}$ – non-zero vector). Vectors $\bar{x}$ satisfying this equality are called eigenvectors of matrix A (or graph G) corresponding to their eigenvalues.

Another common type of graph spectrum is the spectrum of the Laplace matrix L.

The Laplace matrix is used to calculate the tree graphs, as well as to obtain some important spectral characteristics of the graph. Laplace matrix, $L = D - A$ where D — diagonal matrix of order n:

$$d_{ij} = \begin{cases} d_i, \ i = j, \\ 0, \ i \neq j, \end{cases} \tag{2}$$

where d_i — the degree of the corresponding vertex of the graph.

The spectrum $S_{p_L}(G)$ of the matrix L is the root of the characteristic equation

$$|\lambda I - L| = |\lambda I - D + A| = 0. \tag{3}$$

Comparing the spectra S_p, S_{p_L} it is easy to establish that:

$$S_p(G) = [\lambda_1, \lambda_2, ..., \lambda_n],$$
$$S_{p_L}(G) = [r - \lambda_n, r - \lambda_{n-1}, ..., r - \lambda_1],$$

where $\lambda_1 = r$.

The number zero is the eigenvalue of the matrix L, which corresponds to an eigenvector whose coordinates are equal to unity. The multiplicity of the null eigenvalue is equal to the number of connected components of the graph. The rest of eigenvalues L are positive. The least of the positive eigenvalues λ_2 is called the index of algebraic connectivity of the graph. This value represents the «force» of the connectivity of the graph component and is used in the analysis of reliability and synchronization of the graph.

Important derivative characteristics are spectral gap, graph energy, spectral moments and spectral radius. The spectral gap is the difference between the largest and the next eigenvalues of the adjacency matrix and characterizes the rate of return of the system to the equilibrium state. The graph energy is the sum of the modules of the eigenvalues of the graph adjacency matrix:

$$E(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{4}$$

The spectral radius is the largest modulus of the eigenvalue of the adjacency matrix. Denote by N_c the value which corresponds to an «average eigenvalue» of the graph adjacency matrix:

$$N_c = ln\left(\frac{1}{n}\sum_{i=1}^{n} exp(\lambda_i)\right) \qquad (5)$$

and is called natural connectivity.

The k-th spectral moment of the adjacency matrix is determined by the expression:

$$m_k(A) = \frac{1}{n}\sum_{i=1}^{n}\lambda_i^k, \qquad (6)$$

where λ_i is the eigenvalues of the adjacency matrix, n is the number of vertices (nodes) of the graph G.

Among the topological measures one of the most important is the node degree k — the number of links attached to this node. For non-directed networks, the node's degree k_i is determined by the sum $k_i = \sum_j a_{ij}$, where the elements a_{ij} of the adjacency matrix.

To characterize the «linear size» of the network, useful concepts of mean $\langle l \rangle$ and maximum $l_{\max}$ shortest paths. For a connected network of n nodes, the average path length is equal to

$$\langle l \rangle = \frac{2}{n(n-1)}\sum_{i>j} l_{ij}, \qquad (7)$$

where l_{ij} — the length of the shortest path between the nodes. The diameter of the connected graph is the maximum possible distance between its two vertices, while the minimum possible is the radius of the graph.

If the average length of the shortest path gives an idea of the whole network and is a global characteristic, the next parameter — the clustering coefficient — is a local value and characterizes a separate node. For a given node m, the clustering coefficient C_m is defined as the ratio of the existing number of links between its closest neighbors E_m to the maximum possible number of such relationships:

$$C_m = \frac{2E_m}{k_m(k_m-1)}. \qquad (8)$$

In (8) $k_m(k_m-1)/2$ is the maximum number of links between the closest neighbors. The clustering coefficient of the entire network is defined as the average value C_m of all its nodes. The clustering coefficient shows how many of the nearest neighbors of the given node are also the closest neighbors to each other. He characterizes the tendency to form groups of interconnected nodes — clusters. For real-life networks, the high values of the clustering coefficient are high.

Another feature of the node is the betweenness. It reflects the role of the node in establishing network connections and shows how many shortest paths pass through this node. Node betweenness σ_m is defined as

$$\sigma_m = \sum_{i \neq j} \frac{B(i,m,j)}{B(i,j)}, \tag{9}$$

where $B(i,j)$ — the total number of shortest paths between nodes i and j, $B(i,m,j)$ — the number of shortest paths between i,j those passing through the node m. The value (9) is also called the load or betweenness centrality.

One of the main characteristics of the network is the distribution of nodes $P(k)$, which is defined as the probability that the node i has a degree $k_i = k$. For most natural and actual artificial networks there is a power distribution

$$P(k) \sim 1/_{k^\gamma}, k \neq 0, \gamma > 0. \tag{10}$$

Also important topological characteristics are the vertex eccentricity — the largest distance between m and any other vertex, that is, how far the vertex is from the other vertices of the graph. The centrality of the vertex measures its relative importance in the graph. At the same time, the farness of a node is defined as the sum of its distances to all other nodes, and its closeness is defined asthe reciprocal of the farness.

Another important measure is the link density in the graph, which is defined as the existing number of links n_e, divided by the expression $(n - 1)/2$, where n is the number of nodes of the graph.

A multilayer/multiplex network is a pair (G, C) where $G = \{G_\alpha;\ \alpha \in \{1,...,M\}\}$ there is a family of graphs (whether directed or not, weighed or not) $G_\alpha = (X_\alpha, E_\alpha)$, called layers; and

$$C = \{E_{\alpha\beta} \subseteq X_\alpha \times X_\beta;\ \alpha, \beta \in \{1,...,\mathrm{M}\},\ \alpha \neq \beta\}. \tag{11}$$

The latter is a set of links between nodes of different layers G_a and G_β at $\alpha \neq \beta$. Each element E_a is intralayer bonds M in contrast to the elements of each $E_{\alpha\beta}(\alpha \neq \beta)$, called interlayer bonds.

A set of nodes of a layer G_a is denoted $X_\alpha = \{x_1^\alpha, ..., x_{N_\alpha}^\alpha\}$, and a adjacency matrix as $A^{[\alpha]} = \left(a_{ij}^\alpha\right) \in B^{N_\alpha \times N_\alpha}$, where

$$a_{ij}^\alpha = \begin{cases} 1, \left(x_i^\alpha, x_j^\alpha\right) \in E_\alpha, \\ \qquad 0 \end{cases} \tag{12}$$

for $1 \le i, j \le N_\alpha$ and $1 \le \alpha \le M$. For an interlayer adjacency matrix we have $A^{[\alpha,\beta]} = \left(a_{ij}^{\alpha\beta}\right) \in B^{N_\alpha \times N_\beta}$, where

$$a_{ij}^{\alpha\beta} = \begin{cases} 1, \left(x_i^\alpha, x_j^\beta\right) \in E_{\alpha\beta}, \\ \qquad 0 \end{cases} \tag{13}$$

for $1 \le i \le N_\alpha, 1 \le j \le N_\beta$ and $1 \le \alpha, \beta \le M, \alpha \neq \beta$.

A multiplex network is a partial case of interlayer networks and contains a fixed number of nodes connected by different types of links. Multiplex networks are characterized by correlations of different nature [16], which enable the introduction of additional multiplexes.

Let's evaluate the quantitative overlap between the various layers. The average edge overlap equal [28]

$$\omega = \frac{\sum_i \sum_{j>i} \sum_\alpha a_{ij}^{[\alpha]}}{M \sum_i \sum_{j>i} (1 - \delta_{0, \sum a_{ij}^{[\alpha]}})}, \tag{14}$$

and determines the number of layers in which this bond is present. Its value lies on the interval $[1/M, 1]$ and equals $1/M$ if the connection (i, j) exists only in one layer, that is, if there is a layer a such that $a_{ij}^{[\alpha]} = 1,\ a_{ij}^{[\beta]} = 0\ \forall\ \beta \neq \alpha$. If all layers are identical, then $\omega = 1$. Consequently, this measure can serve as a measure of the coherence of the output time series: high values ω indicate a noticeable correlation in the structure of time series.

The total overlap $O^{\alpha\beta}$ between the two layers α and β is defined as the total number of bonds that are shared between the layers α and β:

$$O^{\alpha\beta} = \sum a_{ij}^{\alpha} a_{ij}^{\beta} , \tag{15}$$

where $\alpha \neq \beta$.

For a multiplex network, the vertex degree *k* become a vector

$$k_i = (k_i^{[1]}, ..., \ k_i^{[M]}) , \tag{16}$$

where $k_i^{[\alpha]}$ is the degree of the node in the layer, that is, while the elements of the matrix of adjacency for the layer. Specificity of the vector character of the degree of the peak in multiplex networks allows for the introduction of additional interlayer characteristics. One of these is the overlap of the node's degree

$$o_i = \sum_{\alpha=1}^{M} k_i^{[\alpha]} . \tag{17}$$

The next measure quantitatively describes the interlayer correlations between the degrees of the selected node in two different layers. If, chosen from M the layers of the pair (α, β) characterized by the distribution of degrees $P(k^{[\alpha]}),\ P(k^{[\beta]})$, the so-called interlayer mutual information is determined by the formula:

$$I_{\alpha,\beta} = \sum \sum P(k^{[\alpha]}, k^{[\beta]}) \log \frac{P(k^{[\alpha]}, k^{[\beta]})}{P(k^{[\alpha]}) P(k^{[\beta]})} , \tag{18}$$

where $P(k^{[\alpha]}, k^{[\beta]})$ is the probability of finding a node degree $k^{[\alpha]}$ in a layer α and degree $k^{[\beta]}$ in a layer β. The higher the $I_{a,\beta}$ value, the more correlated are the distributions of the levels of the two layers, and, consequently, the structure of the time series associated with them. We also find the mean value $I_{a,\beta}$ for all possible pairs of layers — the scalar value $\langle I_{a,\beta} \rangle$ that quantifies the information flow in the system.

The quantity that quantitatively describes the distribution of the node degree *i* between different layers is the entropy of the multiplexed degree:

$$S_i = -\sum_{\alpha=1}^{M} \frac{k_i^{[\alpha]}}{o_i} \ln \frac{k_i^{[\alpha]}}{o_i} . \tag{19}$$

Entropy is zero if all nodes are in the same layer and vice versa, has the maximum value when they are evenly distributed between different layers. That is, the higher the value S_i, the even more links evenly the nodes' connections are distributed between the layers.

A similar magnitude is the multiplex participation coefficient:

$$P_i = \frac{M}{M-1}\left[1-\sum_{\alpha=1}^{M}\left(\frac{k_i^{[\alpha]}}{o_i}\right)^2\right]. \qquad (20)$$

P_i takes values on the interval [0,1] and determines that homogeneous links of node i are distributed among M the layers. If all links of the node i lie in one layer, $P_i = 0$ and $P_i = 1$ if the node has a precisely defined number of links in each of the M layers. Consequently, the larger the coefficient P_i is, the more evenly distributed the participation of the node in the multiplex.

Obviously, the magnitudes S_i and P_i are very similar.

We will show that some of these spectral and topological measures serve as measures of complexity of the system, and the dynamics of their changes allows us to build predictors of crisis situations on financial markets.

Analysis of previous publications

Recently, the first papers using the spectral and topological characteristics of dynamic systems presented as networks have appeared. Thus, in [21] it has been investigated universal and nonuniversal allometric scaling behaviors in the visibility graphs of 30 world stock market indices. It has been established that the nature of such behavior is due by the fat-tailedness of the return distribution, the nonlinear long-term correlation, and a coupling effect between these two influential factors.

The author [22] compared the mean degree value and clustering coefficient for a group of companies included in the DAX 30 index basket. He observed the companies from the DAX 30 index for two time periods: the first from the beginning of 2008 through the end of 2009 and the second from the beginning of 2010 up to the end of 2011 as these include the dates — a period of crisis (7th October 2008 — 31st December 2008) and a period of recovery (7th May 2010 —

3rd August 2010). Contrary to expectations, the results differed little from the relatively low accuracy of the HVG procedure compared to VG.

In the work [23], the data of 2571 stock companies in 2012 and the data of 2578 stock companies in 2013 are collected from Chinese stock market. Every year, data of these stock companies are randomly arranged. These data are then converted into some complex networks based on the visibility graph method. For these complex networks, degree distribution and clustering coefficient are considered. These results show that the complex networks have the power-law distribution and small-world properties.

The authors of the article [24] construct an indicator to measure the magnitude of the super-exponential growth of stock prices, by measuring the degree of the price network, generated from the price time series. Twelve major international stock indices have been investigated. The work results show that this new indicator has strong predictive power for financial extremes, both peaks and troughs. By varying the models parameters, authors show the predictive power is very robust. The new indicator has a better performance than the indicator based on a well-known model of log-periodic oscillations of D. Sornette [25].

Authors of another work [26] analyze high frequency data from S&P 500 via the HVG method, and find that all major crises that have taken place worldwide for the last twenty years, affected significantly the behavior of the price-index. Nevertheless, they observe that each of those crises impacted the index in a different way and magnitude. These results suggest that the predictability of the price-index series increases during the periods of crises.

In the work [27] the researchers study the visibility graphs built from the time series of several stock market indices. They propose a validation procedure for each link of these graphs against a null hypothesis derived from ARCH-type modelling of such series. Building on this framework, made it possible to devise a market indicator that turns out to be highly correlated and even predictive of financial instability periods.

Multiplex networks are actively used to simulate complex networks of different nature: from financial (stock market [26–29], banks [30], guarantee market [31]) to social [32]. Particular attention should be paid to the work [29], in which the above multiplex measures are analyzed for the subject of correlations with known stock markets crises.

Yet there is no systematic analysis of network and multiplex measures and the construction of indicators-predictors of the crisis phenomena in the stock market.

Experimental results and their discussion

The time series of daily values of stock market indexes for the period from 01/01/1983 to 10/01/2019 were selected as databases, which contained significant changes in the indexes, and were identified as crisis phenomena [33]. Among the set of stock indexes are the following:

SP (S&P 500) — USA;
FCHI (CAC 40) — France;
DAX (DAX PERFORMANCE-INDEX) — Germany;
N225 (Nikkei 225) — Japan;
HSI (HANG SENG INDEX) — China;
BSESN (S&P BSE SENSEX) — India;
KS11 (KOSPI Composite Index) — South Korea;
GSPTSE (S&P / TSX Composite index) — Canada;
BVSP (IBOVESPA) — Brazil.

Since historical intervals in storing stock indexes are different, we have formed two databases. One of them includes only three indices, but since 1983. The next is already 9 daily index values, but for a shorter period of time — since 2004 (Fig. 3).

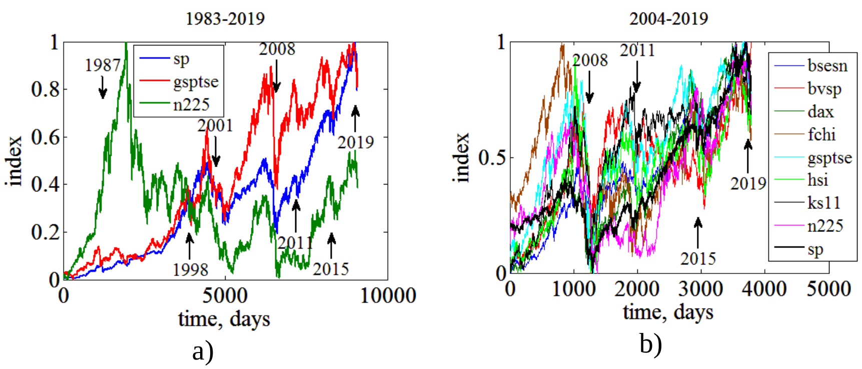

Fig. 3. Dynamics of daily values of stock indexes (a) of the USA, Canada and Japan during 1983–2019 and (b) the aggregate of all the considered indices for the period 2004–2019

The first short database allows you to analyze the seven most well-known crises (shown in Fig. 3.a), while the second one is only 4 (Fig. 3.b).

Calculations of spectral and topological measures by methods of VG, HVG were carried out in the following way. The time window was chosen, for example, a year or two (approximately 250 or 500 trading days), for which the corresponding graphs were constructed and their spectral, topological and multiplex properties were calculated. Next, the window was shifted step by step, for example, one week (5 trading days) and the procedure repeated until the time series were exhausted.

The results of calculations for revived time series of graphs are shown in Figs. 4–6. Knowing the time of the onset of the crisis and comparing the time series with the dynamics of a certain indicator, it is possible to investigate its dependence on certain or other characteristic changes in the stock market: pre-crisis, crisis and post-crisis periods.

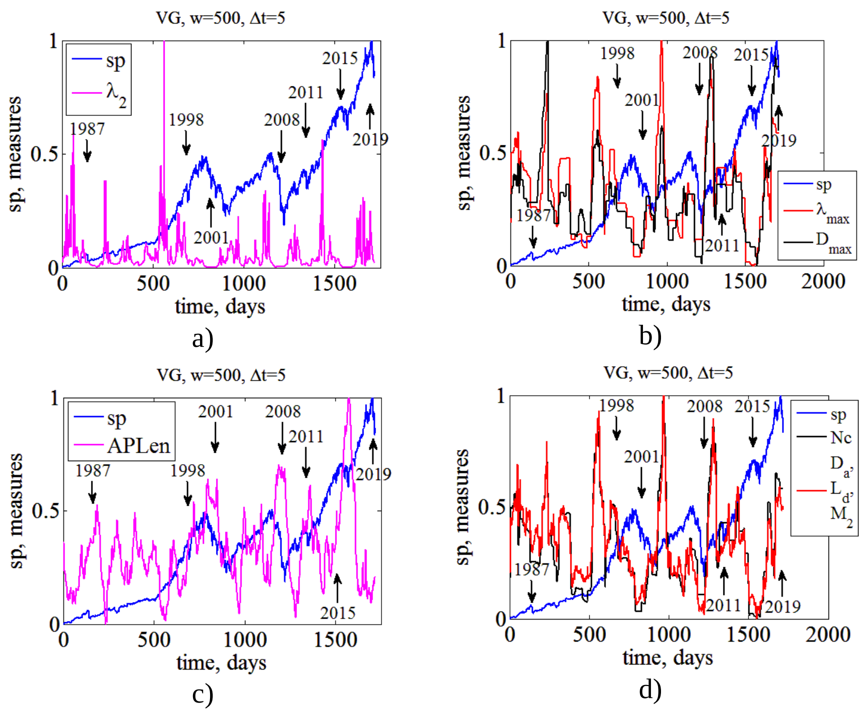

Fig. 4. Dynamics of the S&P 500 index and the spectral (a), (b), topological (c) and spectral with the topological (d) measure of the network constructed by the visibility graph algorithm

From the spectral measures, we consider it important to predict the algebraic connectivity (λ_2 — Fig. 4.a), the maximal eigenvalues of the adjacency matrix (spectral radius) and the maximal node degree (λ_{max} and D_{max} — Fig. 4.b). From the topological measures, the average path length (APLen — Fig. 4.c) is found, which is in accordance with formula (7). Fig. 4.d demonstrates the universality of the spectral behavior (the graph natural connectivity N_c and the second spectral moment of the adjacency matrix M_2) and the topological (mean node degree D_a and link density L_d).

Figure 4 shows that all of the above spectral measures have maximum values in pre-crisis periods. The complex system has the greatest complexity. With the approach of the crisis, the complexity of the system decreases, recovering from the crisis. Some of the topological, in particular, APLen, the diameter of the graph, etc., show an opposite relationship. Indeed, in more complex systems you can always find shorter paths that connect any nodes. During the crisis (reducing complexity, increasing the chaotic component), the length of the corresponding path increases.

Parameters such as the width of the window w and the step of its displacement along the time series are important. When w is small, the degree of complexity fluctuates noticeably, reacting not only to crises, but also to more or less noticeable fluctuations of the index. On the contrary, with too much window width there is a noticeable smoothing of the appropriate measure and if two crises are at a distance that is smaller than w, the indicators of both crises are averaged and less informative. If you choose an oversized parameter Δt, you might miss the actual crisis that distorts the indicator.

As far as multiplex measures are concerned, they are very similar in their dynamics to the spectral and topological representations above (see Fig. 5). In the case of a shorter sample of a base of three layers (Figs. 5.a, 5.b), we have the antisymmetric behavior of the multiplex measures O, o, I (formulas (15), (17), (18)) and S, P (formulas (19), (20)). A similar, albeit more noisy picture is observed in the case of a shorter observation time, but with 9 layers of base (Figs. 5.c, 5.d).

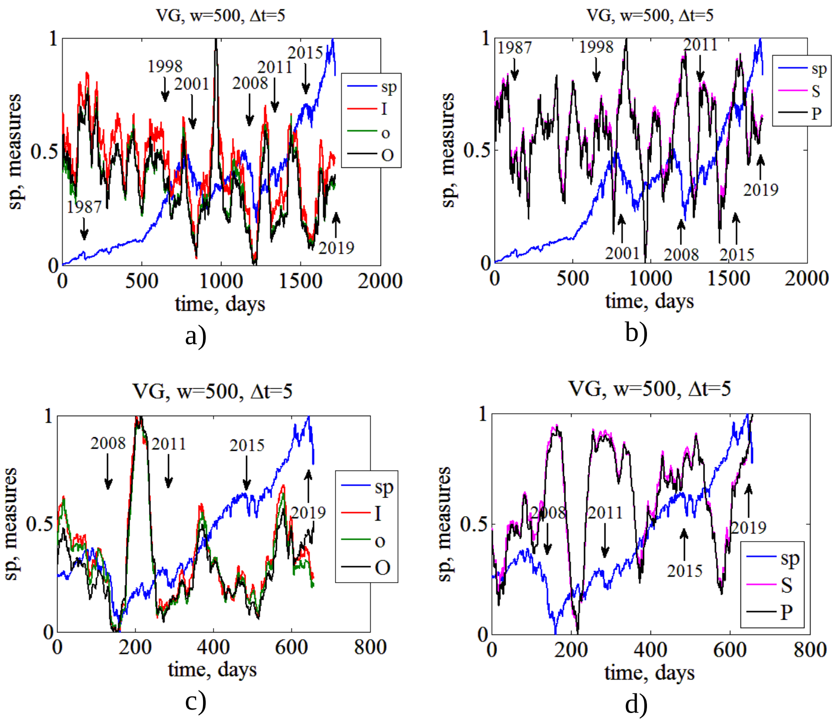

Fig. 5. Dynamics of S&P 500 index and multiplex measures
for a base of three (a), (b) and nine layers (c), (d).
The graph was built using the multiplex visibility graph

Figures 5 show that both multiplex measures are excellent indicators that warn in advance about the approaching crisis phenomenon, that is, are indicator-predictor.

The MVG algorithm does not fundamentally change the picture, but predictive indicators are not as clear as in the case of VG. Fig. 6.a shows this conclusion on the example of the spectral radius, and Figs. 6.c, 6.d — multiplex measures. Fig. 6.b shows the immutability of the dynamics of the spectral measure (on an example of algebraic connectivity) with a decrease in the width of a moving window from 500 to 250 days.

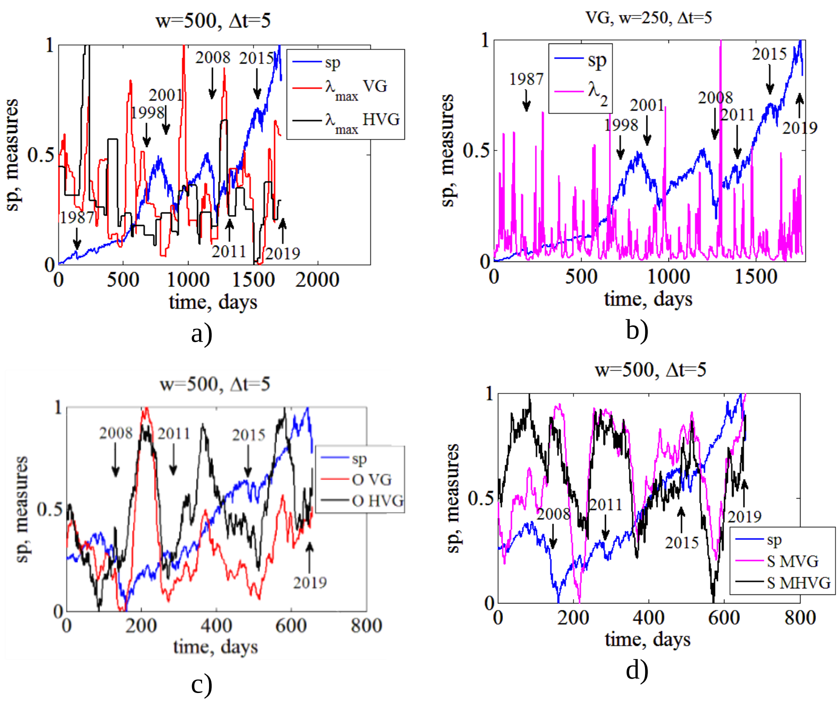

Fig. 6. Dynamics of the S&P 500 index and the spectral and multiplex complexity measures, calculated on the basis of algorithms VG, MVG, MHVG

Conclusions

Thus, we have demonstrated the possibility of studying complex socio-economic systems as part of a network paradigm of complexity. A time series can be represented in an equivalent way — a network, or a multiplex network, which has a wide range of characteristics; both spectral and topological, and multiplexed. Examples of known financial crises have shown that most of the network measures can serve as indicators-precursors of crisis phenomena and can be used for possible early prevention of unwanted crises in the financial markets. They are an extension of the already proposed by us and «working» indicators, which use other measures of complexity [34].

It should be noted that the proposed indicators-precursors do not solve the more general problem of forecasting future values or trends of the stock market. In this way, it is possible to use new approaches (see, for example, [35, 36]) or alternative methods based on algorithms of (deep) machine learning [37].

Список літератури

1. The Global Risks Report 2019. *World Economic Forum*. 2019. URL: http://www3.weforum.org/docs/WEF_Global_Risks_Report_2019.pdf.

2. Roubini N., Rosa B. The Makings of a 2020 Recession and Financial Crisis. 2018. URL: https://www.project-syndicate.org/commentary/financial-crisis-in-2020-worse-than-2008-by-nouriel-roubini-and-brunello-rosa-2018-09?barrier=accesspaylog.

3. Малинецкий Г. Г. Теория самоорганизации. На пороге IV парадигмы. *Компьютерные исследования и моделирование*. 2013. Т. 5, № 3. С. 315–366.

4. Prigogine I. From Being to Becoming Time and Complexity in the Physical Sciences. New York: W. H. Freeman & Company, 1980. 272 p.

5. Barrat A., Barthelemy M., Vespignani A. Dynamical processes on complex networks. Boston: Cambridge University Press, 2008. 347 p.

6. Cohen R., Havlin S. Complex networks. Structure, robustness and function. New York: Cambridge University Press, 2010. 248 p.

7. Albert R., Barabasi A.-L. Statistical Mechanics of Complex Networks. *Reviews of Modern Physics*. 2002. Vol. 74. P. 47–97.

8. Newman M., Watts D., Barabási A.-L. The Structure and Dynamics of Networks. Princeton: Princeton University Press, 2006. 456 p.

9. Newman M. E. J. The structure and function of complex networks. *SIAM Reviews*. 2003. Vol. 45, No. 2. P. 167–256.

10. Boccaletti S., Latora V., Moreno Y., Chavez M., Hwang D.-U. Complex networks: Structure and dynamics. *Physics Reports*. 2006. Vol. 424, Is. 4–5. P. 175–308.

11. Bianconi G. Interdisciplinary and physics challenges in network theory. *Europhysics Letters*. 2015. Vol. 111, № 5. P. 1–7.

12. Danilchuk G., Soloviev V. Dynamics of graph spectral entropy in financial crisis. *Socio-economic aspects of economics and management*. 2015. Vol. 2. P. 227–234.

13. Марш П. Новая промышленная революция. Потребители, глобализация и конец массового производства / Пер. с англ. А. Шоломицкой. — М.: Изд-во Института Гайдара, 2015. 420 с.

14. Соловйов В. М., Соловйова К. В. Кількісні методи оцінки складності в прогнозуванні соціально-економічних систем. Прогнозування

соціально-економічних процесів: сучасні підходи та перспективи: монографія / За ред. О. І. Черняка, П. В. Захарченка. Бердянськ: Видавець Ткачук О. В., 2012. 542 с.

15. Соловйов В. М. Мережні міри складності соціально-економічних систем. *Вісник Черкаського університету. Серія «Прикладна математика. Інформатика»*. 2015. № 38 (371). С. 67–79.

16. Boccaletti S., Bianconi G., Criado R., delGenio C. I., Gómez-Gardeñes J., Romance M., Sendiña-Nadal I., Wang Z., Zanin M. The structure and dynamics of multilayer networks. *Physics Reports*. 2014. Vol. 544, No. 1. P. 1–122.

17. Donner R. V., Small M., Donges J. F., Marwan N., Zou Y., Xiang R., Kurths J. Recurrence-based time series analysis by means of complex network methods. *International Journal of Bifurcation and Chaos*. 2011. Vol. 21, No. 4. P. 1019–1046.

18. Соловйов В. М., Тулякова А. Ш. Графодинамічні методи дослідження складності сучасних фондових ринків. *Нейро-нечіткі технології моделювання в економіці*. 2016. № 5. С. 152–179.

19. Lacasa L., Luque B., Ballesteros F., Luque J., Nuño J.C. From time series to complex networks: The visibility graph. *PNAS*. 2008. Vol. 105, No. 13. P. 4972–4975.

20. Luque B., Lacasa L. Ballesteros F., Luque J. Horizontal visibility graphs: Exact results for random time series. *Physical Review E*. 2009. Vol. 80, Iss. 4. P. 1–11.

21. Qian M. C., Jiang Z. Q., Zhou W. X. Universal and nonuniversal allometric scaling behaviors in the visibility graphs of world stock market indices. *Journal of Physics A: Mathematical and Theoretical*. 2010. Vol. 43, No. 33. P. 1–12.

22. Birch J. L. *Modelling Financial Markets using Methods from Network Theory*: Thesis degree of Doctor in Philosophy. Liverpool, 2015. 132 p. URL: https://livrepository.liverpool.ac.uk/2028739/1/BirchJen_Aug2015_2028739.pdf.

23. Liu F., Wang N., Wei D. Analysis of Chinese Stock Market by Using the Method of Visibility Graph. *The Open Cybernetics & Systemics Journal*. 2017. Vol. 11. P. 36–43.

24. Yan W., Serooskerken E. T. Forecasting Financial Extremes: A Network Degree Measure of Super-Exponential Growth. *PLoSONE*. 2015. Vol. 10, No. 9. P. 1–15.

25. Johansen A., Ledoit O., Sornette D. Crashes as critical points. *International Journal of Theoretical and Applied Finance*. 2000. Vol. 3, No. 2. P. 219–255.

26. Vamvakaris M. D., Pantelous A. A., Zuev K. M. Time series analysis of S&P 500 index: A horizontal visibility graph approach. *Physica A*. 2018. Vol. 497. P. 41–51.

27. Serafino M., Gabrielli A., Cardarelli G., Cimini G. Statistical validation of financial time series via visibility graph. 2017. P. 1-9. URL: arXiv:1710.10980v1.

28. Nicosia V., Lacasa L., Latora V. Network structure of multivariate time series. *Scientific Reports*. 2015. Vol. 5. P. 1–7.

29. Musmeci N., Nicosia V., Aste T., Di Matteo T., Latora V. The multiplex dependency structure of financial markets. *Complexity*. 2017. P. 1–13.

30. Bargigli L., di Iasio G., Infante L., Lillo F., Pierobon F. The multiplex structure of interbank networks. *Quantitative Finance*. 2015. Vol. 15. P. 673–691.

31. Li S., Wen S. Multiplex Networks of the Guarantee Market: Evidence from China. *Complexity*. 2017. P. 1–7.

32. Stephen C. Dynamic Phase and Group Detection in Pedestrian Crowd Data Using Multiplex Visibility Graphs. *Procedia Computer Science*. 2015. Vol. 53, No. 1. P. 410–419.

33. Stock Market. *Yahoo Finance* : website. URL: http://finance.yahoo.com (Last accessed: 11.01.2019).

34. Soloviev V., Belinskij A. Methods of nonlinear dynamics and the construction of cryptocurrency crisis phenomena precursors. *Proceedings of the 14th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer*. Kyiv, 2018. Vol. II. P. 116–127. URL: http://ceur-ws.org/Vol-2104/paper_175.pdf.

35. Soloviev V., Saptsin V., Chabanenko D. Financial time series prediction with the technology of complex Markov chains. *Computer Modelling and New Technologies*. 2010. Vol. 14, No. 3. P. 63–67.

36. Soloviev V., Saptsin V., Chabanenko D. Markov Chains application to the financial-economic time series prediction. *Computer Modeling and New Technologies*. 2011. Vol. 15, No. 3. P. 16–20. URL: arXiv:1111.5254v1.

37. Chatzis S. P., Siakoulis V., Petropoulos A., Stavroulakis E., Vlachogiannakis N. Forecasting stock market crisis events using deep and statistical machine learning techniques. *Expert Systems With Applications*. 2018. Vol. 112. P. 353–371.

References

1. World Economic Forum. (2019). *The Global Risks Report 2019.* Retrieved from http://www3.weforum.org/docs/WEF_Global_Risks_Report_2019.pdf.

2. Roubini, N., & Rosa, B. (2018). *The Makings of a 2020 Recession and Financial Crisis*. Retrieved from https://www.project-syndicate.org/commentary/financial-crisis-in-2020-worse-than-2008-by-nouriel-roubini-and-brunello-rosa-2018-09?barrier=accesspaylog.

3. Malinetskiy, G. G. (2013). Teoriya samoorganizatsii. Na poroge IV paradigmy. *Kompyuternyye issledovaniya i modelirovaniye (Computer research and modeling), 5*(3), 315–366 [in Russian].

4. Prigogine, I. (1980). *From Being to Becoming Time and Complexity in the Physical Sciences*. New York, NY: W.H. Freeman & Company.

5. Barrat, A., Barthelemy, M., & Vespignani, A. (2008). *Dynamical processes on complex networks*. Boston, MA: Cambridge University Press. DOI: 10.1017/CBO9780511791383.

6. Cohen, R., & Havlin, S. (2010). *Complex networks. Structure, robustness and function*. New York, NY: Cambridge University Press. DOI: 10.1017/CBO9780511780356.

7. Albert, R., & Barabasi, A.-L. (2002). Statistical Mechanics of Complex Networks. *Reviews of Modern Physics, 74*, 47–97. DOI: 10.1103/RevModPhys.74.47.

8. Newman, M., Watts, D., & Barabási, A.-L. (2006). *The Structure and Dynamics of Networks*. Princeton, NJ: Princeton University Press.

9. Newman, M. E. J. (2003). The structure and function of complex networks. *SIAM Reviews, 45*(2), 167–256. DOI: 10.1137/S003614450342480.

10. Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., & Hwang, D.-U. (2006). Complex networks: Structure and dynamics. *Physics Reports, 424*(4–5), 175–308.

11. Bianconi, G. (2015, September 1). Interdisciplinary and physics challenges in network theory. *Europhysics Letters, 111*(5), 1–7. DOI: 10.1209/0295-5075/111/56001.

12. Danilchuk, H., & Soloviev, V. (2015). Dynamics of graph spectral entropy in financial crisis. *Socio-economic aspects of economics and management, 2*, 227–234.

13. Marsh, P. (2012). *The New Industrial Revolution: Consumers, Globalization and the End of Mass Production*. London, UK: Yale University Press.

14. Soloviev, V. M., & Solovieva, K. V. (2012). Kil'kisni metody otsinky skladnosti v prognozuvanni sotsialno-ekonomichnykh system. In O. I. Chernyak & P. V. Zakharchenko (Eds.), *Prognozuvannya sotsial'no-ekonomichnykh protsesiv: suchasni pidkhody ta perspektyvy* (pp. 170–184). Berdyansk, Ukraine: Vidavets' Tkachuk O. V. [in Ukrainian].

15. Soloviev, V. N. (2015). Merezhni miry skladnosti sotsial'no-ekonomichnykh system. *Visnik Cherkas'kogo universitetu. Seriya «Prikladna matematika. Informatika» (Bulletin of the Cherkasy University. Series «Applied Mathematics. Computer Science»), 38*(371), 67–79 [in Ukrainian].

16. Boccaletti, S., Bianconi, G., Criado, R., delGenio, C.I., Gómez-Gardeñes, J., Romance, M., Sendiña-Nadal, I., Wang, Z., & Zanin, M. (2014). The structure and dynamics of multilayer networks. *Physics Reports, 544*(1), 1–122. DOI: 10.1016/j.physrep.2014.07.001.

17. Donner, R. V., Small, M., Donges, J. F., Marwan, N., Zou, Y., Xiang, R., & Kurths, J. (2010). Recurrence-based time series analysis by means of complex network methods. *International Journal of Bifurcation and Chaos, 21*(4), 1019–1046. DOI: 10.1142/S0218127411029021.

18. Soloviev, V. N., & Tuliakova, A. Sh. (2016). Grafodynamichni metody doslidzhennya skladnosti suchasnykh fondovykh rynkiv. *Nejro-nechitki tekhnolohii modeliuvannia v ekonomitsi (Neuro-Fuzzy Modeling Techniques in Economics), 5,* 152–179 [in Ukrainian].

19. Lacasa, L., Luque, B., Ballesteros, F., Luque, J., & Nuño, J. C. (2008, April 1). From time series to complex networks: The visibility graph. *PNAS, 105*(13), 4972–4975. DOI: 10.1073/pnas.0709247105.

20. Luque, B., Lacasa, L. Ballesteros, F., & Luque, J. (2009). Horizontal visibility graphs: Exact results for random time series. *Physical Review E, 80*(4), 1–11. DOI:10.1103/PhysRevE.80.046103.

21. Qian, M. C., Jiang, Z. Q., & Zhou, W. X. (2010). Universal and nonuniversal allometric scaling behaviors in the visibility graphs of world stock market indices. *Journal of Physics A: Mathematical and Theoretical, 43*(33), 1–12. DOI: 10.1088/1751-8113/43/33/335002.

22. Birch, J. L. (2015). *Modelling Financial Markets using Methods from Network Theory: PhD Thesis*. Liverpool, UK: University of Liverpool. Retrieved from https://livrepository.liverpool.ac.uk/2028739/1/BirchJen_Aug2015_2028739.pdf.

23. Liu, F., Wang, N., & Wei, D. (2017). Analysis of Chinese Stock Market by Using the Method of Visibility Graph. *The Open Cybernetics & Systemics Journal, 11*, 36–43.

24. Yan, W., & Serooskerken, E. T. (2015, September 4). Forecasting Financial Extremes: A Network Degree Measure of Super-Exponential Growth. *PLoSONE, 10*(9), 1–15. DOI:10.1371/journal.pone.0128908.

25. Johansen, A., Ledoit, O., & Sornette, D. (2000). Crashes as critical points. *International Journal of Theoretical and Applied Finance, 3*(2), 219–255. DOI: 10.1142/S0219024900000115.

26. Vamvakaris, M. D., Pantelous, A. A., & Zuev, K. M. (2018). Time series analysis of S&P 500 index: A horizontal visibility graph approach. *Physica A, 497,* 41–51. DOI: 10.1016/j.physa.2018.01.010.

27. Serafino, M., Gabrielli, A., Cardarelli, G., & Cimini, G. (2017, October 30). Statistical validation of financial time series via visibility graph, 1–9. Retrieved from arXiv:1710.10980v1.

28. Nicosia, V., Lacasa, L., & Latora, V. (2015). Network structure of multivariate time series. *Scientific Reports, 5*, 1–7. DOI: 10.1038/srep15508.

29. Musmeci, N., Nicosia, V., Aste, T., Di Matteo, T., & Latora, V. (2017, September 20). The multiplex dependency structure of financial markets. *Complexity*, 1–13. DOI: 10.1155/2017/9586064.

30. Bargigli, L., di Iasio, G., Infante, L., Lillo, F., & Pierobon, F. (2015). The multiplex structure of interbank networks. *Quantitative Finance, 15,* 673–691.

31. Li, S., & Wen, S. (2017, July 9). Multiplex Networks of the Guarantee Market: Evidence from China. *Complexity*, 1–7. DOI: 10.1155/2017/9781890.

32. Stephen, C. (2015). Dynamic Phase and Group Detection in Pedestrian Crowd Data Using Multiplex Visibility Graphs. *Procedia Computer Science, 53*(1), 410–419. DOI: 10.1016/j.procs.2015.07.318.

33. Yahoo Finance. (2019). *Stock Market*. Retrieved from https://finance.yahoo.com.

34. Soloviev, V., & Belinskij, A. (2018, May 14–17). Methods of nonlinear dynamics and the construction of cryptocurrency crisis phenomena precursors. In V. Ermolayev, M. C. Suárez-Figueroa et al. (Eds.), *Proceedings of the 14th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer (Kyiv, Ukraine)* (pp. 116–127). Retrieved from http://ceur-ws.org/Vol-2104/paper_175.pdf.

35. Soloviev, V., Saptsin,V., & Chabanenko, D. (2010). Financial time series prediction with the technology of complex Markov chains. *Computer Modelling and New Technologies, 14*(3), 63–67.

36. Soloviev, V., Saptsin, V., & Chabanenko, D. (2011). Markov Chains application to the financial-economic time series prediction. *Computer Modeling and New Technologies, 15*(3), 16–20. Retrieved from arXiv:1111.5254v1.

37. Chatzis, S. P., Siakoulis, V., Petropoulos, A., Stavroulakis, E., & Vlachogiannakis, N. (2018). Forecasting stock market crisis events using deep and statistical machine learning techniques. *Expert Systems With Applications, 112*, 353–371. DOI: 10.1016/j.eswa.2018.06.032.

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